The Ulam-Hammersley problem for integer sequences and partial orders

A set of integers is, informally, called heapable if its elements can be inserted into a binary tree (not necessarily complete) that respects the heap property. The definition naturally extends to partial orders.

We investigate the partitioning of partial orders into a minimal number of heapable subsets. We prove a characterization result reminiscent of the proof of Dilworth's theorem, which yields as a byproduct a flow-based algorithm for computing such a minimal decomposition.

On the other hand:

- for interval partial orders a longest heapable subsequence can be computed in polynomial time.
- for trapezoid partial orders we prove that a minimal decomposition can be computed by a simple greedy-type algorithm.

The talk will also discuss a couple of open problems related to the analog of the Ulam-Hammersley problem for decompositions of sets and sequences of elements of a partial order into heapable subsets.